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External task

Calculate potential of a single vortex analytically

Motivation:

In order to calculate pressure in vortex-based CFD methods using Cauchy-Lagrange integral, one must know velocity potential, created by vortices at the surface of a body. For a triangular vortex frame, an analytical expression of the potential is known and has relatively simple form. Use of frame-based potentials only severely limits accessible vortex configurations. The problem can be solved by using potential of a single vortex. According to claims in literature, it can be expressed through elementary functions, but it is very complicated and could not be found in existing publications.

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Task formulation:
Find scalar potential of a vector field, defined by

$$\mathbf{F} = \frac{1}{4\pi\sqrt{r^2 - (\mathbf{r}\cdot\mathbf{n})^2}} \left(\frac{(\mathbf{r} + L\mathbf{n})\cdot\mathbf{n}}{|\mathbf{r} + L\mathbf{n}|} - \frac{(\mathbf{r} - L\mathbf{n})\cdot\mathbf{n}}{|\mathbf{r} - L\mathbf{n}|} \right) \frac{\mathbf{r}\times\mathbf{n}}{|\mathbf{r}|}$$

Where ***n*** is a constant unit vector, *L* – positive number, r is a point in space.

The resulting potential *φ(**r**)* should be presented in three forms:

1) Human-readable analytical expression (LaTeX, MSWord).
2) Worksheet, readable by symbolic calculation software (Any open-source (like Sage) is OK, if proprietary – Mathematica is preferred).
3) C++ function.

Hint: The resulting potential is likely to be axially ambiguous.